Engineering With Nature

Lecture 3

Goals of Today's Lecture

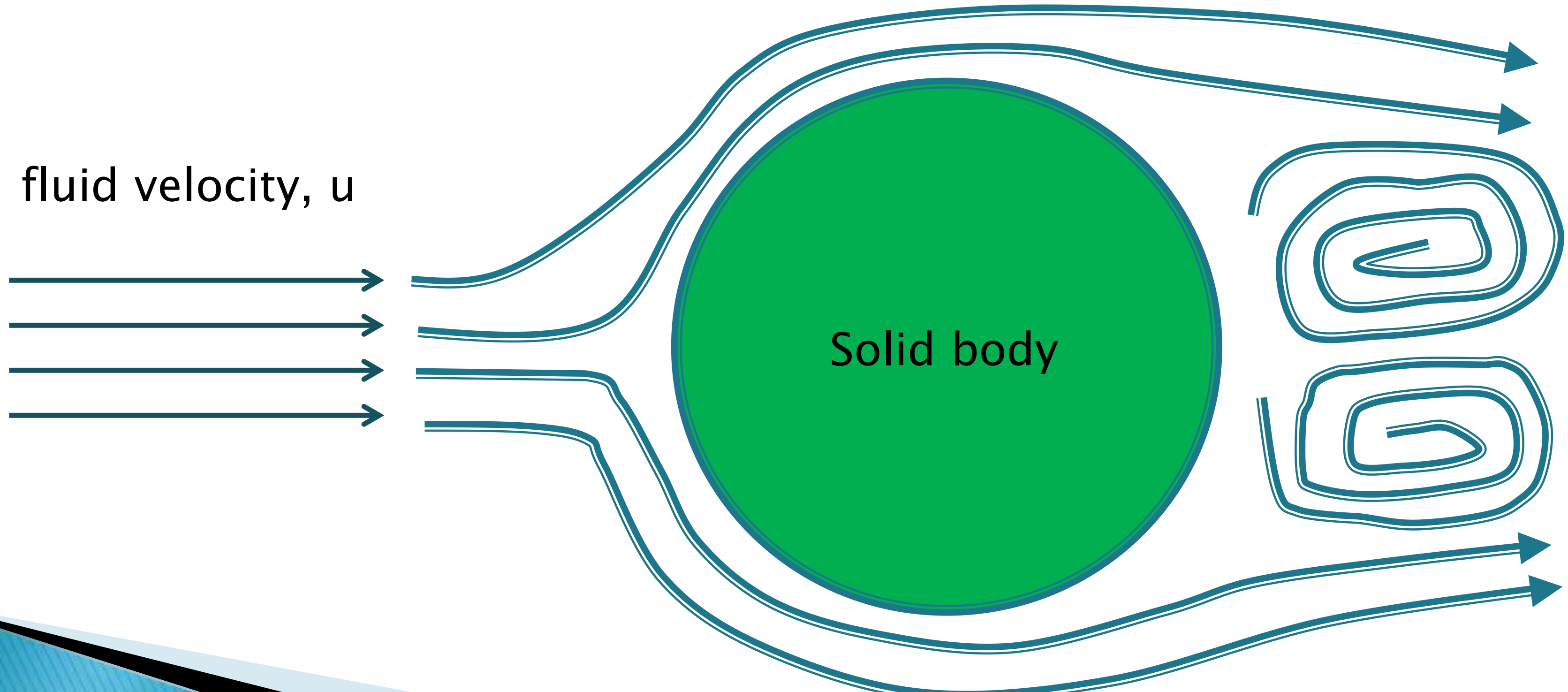
Provide an overview/introduction of

-Drag with Consideration for Vegetation
-Open Channel Flow
-Importance of Froude Number
-Mixing in Vegetation

Drag Force

There are two components of drag

- Skin Friction Drag – Results from the no-slip boundary condition around an object which results in the formation of a boundary layer
- Form Friction Drag – Results from deformation of fluid streamlines around the object which results in fluid accelerations



https://www.youtube.com/watch?v=30_aADFVL9M

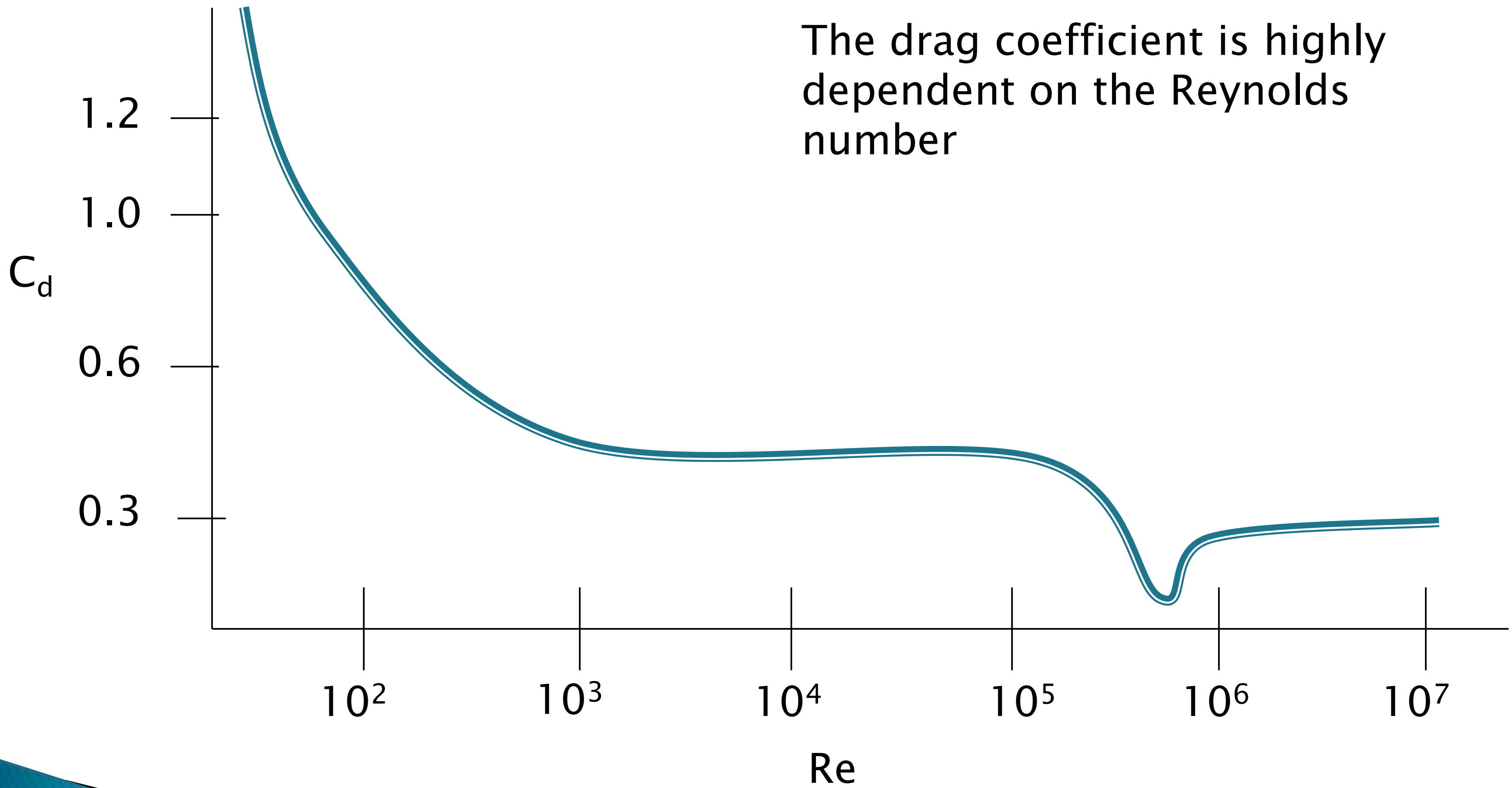
Drag Force

- $F_D = \rho * (1/2) * C_d * A * u^2$, ρ is the density of the fluid (kg/m^3), A is the projected area (m^2), and u is the fluid velocity (m/s)
- What is C_d?
- C_d is the coefficient of drag that is determine experimentally or by computational models
- C_d ultimately represents our inability to quantify directly why some objects have different drag forces even though they have the same projected area, *A*, for a given velocity.

Drag Force

- Values for Coefficient of drag depend on the Reynolds number.
- Remember the Reynolds number is the ratio the convective acceleration (inertial) forces to the viscous forces
- $Re = U*D/\nu$, where U is the representative velocity (m/s), D is the representative length scale (m), and ν is the kinematic viscosity (m^2/s)
- The kinematic viscosity is dependent on temperature but for this course we will assume the value used for 20°C of ($1*10^{-6}$ m^2/s)

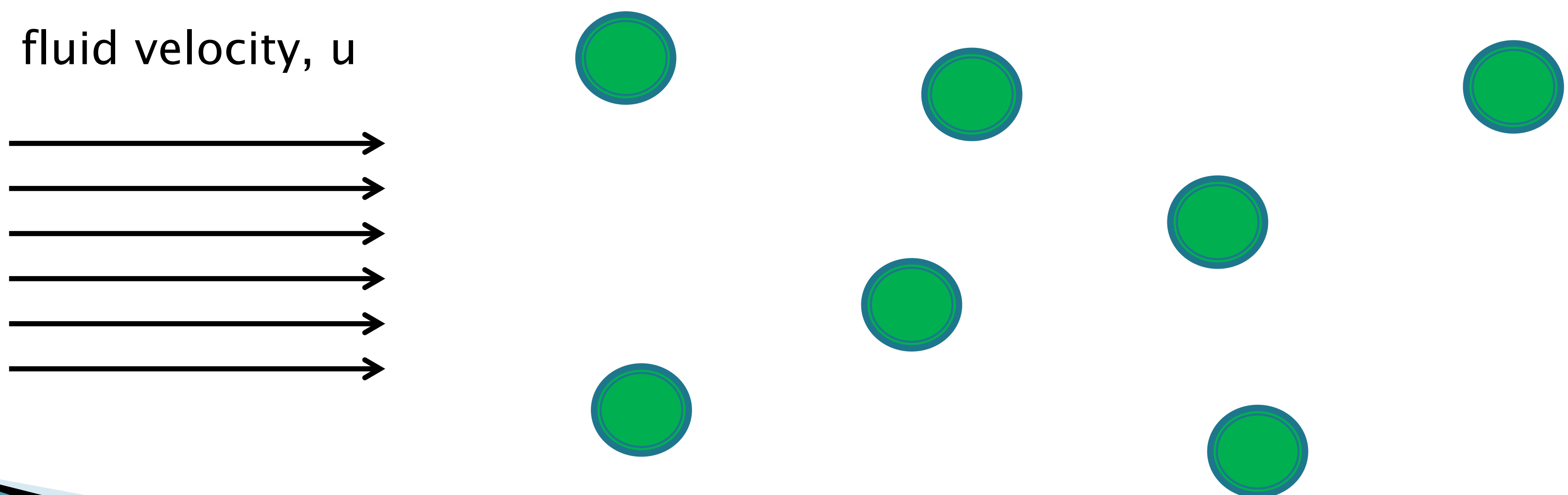
Drag Force



The drag coefficient is highly dependent on the Reynolds number

Drag Force

- The previous equation works great for a single object, but what about multiple objects (multiple stems/shrubs/trees/leaves)



Drag Force

- Can the vegetation withstand the drag imparted by the flow?
- If the vegetation is damaged intermittently will it recover? (Trees/shrubs/grasses/ perennials/annuals)
- How do we relate the vegetation back onto the flow? (Use the momentum equation)
- How do we relate that to a large area with hundreds/thousands/millions of friction elements.
- What if the vegetation is highly flexible?

https://www.youtube.com/watch?v=pS22LfUSVjk

Open Channel Flow

Lecture 2 demonstrated that fluid flows can be modeled using a system of nonlinear partial differential equation

The practical value of these are limited to computational models

Engineers have devised a number of empirical or semi-empirical equations to solve problems without the use of computational models

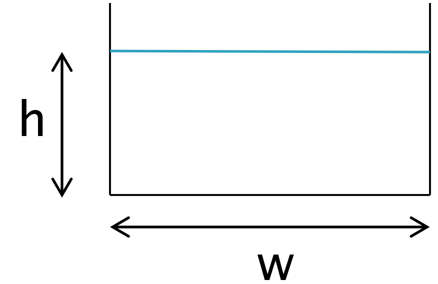
Open Channel Flow

- For a given water depth, *h*, in a channel (river, stream, etc.) how much water, discharge *Q*, is flowing.
- Relevant variables?
  - Geometry
  - Slope – gravity is the driving force
  - Friction

First lets define the Geometric parameters that are important.

We know from the continuity equation that the area, *A,* is important

Another term that is important is the wetted perimeter, *P*



$A = h*w$

$P = w+(2*h)$

Manning Formula

- $Q=(k/n) * A*(R_h^{(2/3)})*(S^{(1/2)})$
  - Where k =1 for SI units, 1.49 for English units
  - A is the cross sectional area
  - R_h is the hydraulic radius, equal to the area, A, divided by the wetted perimeter, P.
  - S is the bed slope
  - n is the manning coefficient

- The manning coefficient has been found empirically
- Data is available for manning coefficient for vegetated environments
- Concrete channels have a manning n ranging from 0.011 to 0.014
- Floodplains have a manning n ranging from 0.035 to 0.160
- Velocity = Flow/Area

Mannings Equation

- Since the Mannings Equation has been used for for through vegetation, there are empirical n values for different types of vegetation.
- Since the manning coefficient is empirically found it accounts for all the problems that make extend the drag equation between one/few elements
- A useful tool but often not used beyond a base level assessment or man-made channels.

Froude Number

- $Fr=u/((g*D)^{(1/2)})$ where u is the mean velocity, g is the gravitational acceleration and D is the hydraulic depth. D=Area/Top Width
- For a rectangular channel $Fr=u/((g*h)^{(1/2)})$ where u is the mean velocity, g is the gravitational acceleration and h is the water depth.
- $Fr > 1$, flow is supercritical, upstream control
- $Fr = 1$, flow is critical
- $Fr < 1$, flow is subcritical, downstream control

Froude Number

- Froude Number gives you an idea into the relative energy in the flow. (Ratio of the convective acceleration to the gravitational force)
- A broad generalization is vegetation will not be sustainable in flow with Froude number equal to or greater than 1. Typical rivers with vegetated islands or floodplains will have Fr much less than 1.

What are we missing?

- Mixing, turbulent and a solution for multiple elements
- $\phi \approx (\pi/4)\,ad$, canopy density equation
- where $a = d/\Delta S^2$, a = frontal area, d=stem diameter, and ΔS is the average spacing between stems
- Spatially average drag associated with canopy elements
- $D_x=(1/2)*(C_d*a/(1-\phi))*u^2$