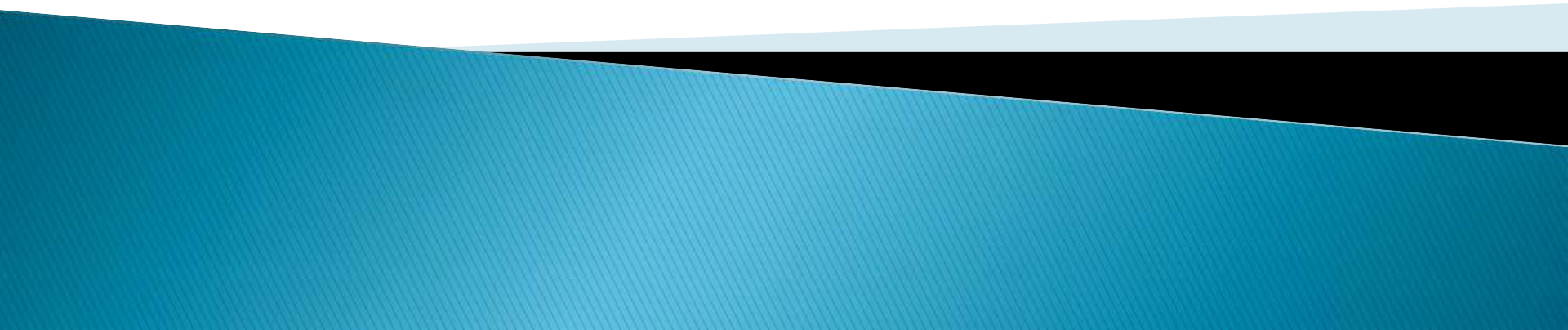
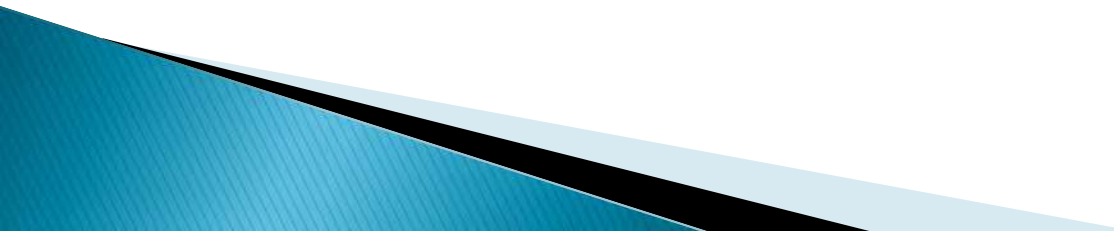


# Engineering With Nature

Lecture 7



# Goals of Today's Lecture

- Shear Stress Calculations for Currents
  - Shear Stress Calculation for Waves
  - Shear Stress Calculation for Combined Currents and Waves
- 

# Defining the Boundary Layer- Currents

- ▶ The current velocity at any distance from the bed,  $z$ , (free stream) can be defined by the current shear velocity,  $u_{*c}$ ,  $\kappa$ , and  $z_o$  the distance above the bed where the velocity is zero.

$$u_c = \frac{u_{*c}}{\kappa} \ln \frac{z}{z_o}$$

- ▶ The shear velocity,  $u_{*c}$ , is defined by the equation where  $\tau_c$  is the current shear stress and  $\rho$  is the fluid density.

$$u_{*c} = (\tau_c / \rho)^{1/2}$$

# Defining $z_0$

- ▶  $z_0$  can be defined by the following equations where  $\nu$  is the viscosity and  $k_n$  is the equivalent sand grain roughness

$$z_0 = \begin{cases} \frac{\nu}{9u_*} & \text{for smooth turbulent flow} \\ \frac{k_n}{30} & \text{for fully rough turbulent flow} \end{cases}$$

# Smooth or Rough Turbulent

- ▶ Determining the flow assumes you know the shear velocity. Thus solving these equations is an iterative process

$$\frac{\kappa_n u_*}{V} \geq 3.3 \quad \text{for fully rough turbulent flow}$$

$$\frac{\kappa_n u_*}{V} \leq 3.3 \quad \text{for fully smooth turbulent flow}$$

# Shear Stress – Current

- ▶ The shear stress is often defined as where  $z_r$  is the reference elevation

$$\tau_c = \frac{1}{2} f_c \rho (u_c(z_r))^2$$

- ▶ Where the friction factor is defined as
- ▶ for fully smooth turbulent flow

$$\frac{1}{4\sqrt{f_c}} + \log_{10} \frac{1}{4\sqrt{f_c}} = \log_{10} \frac{z_r u_c(z_r)}{\nu} + 0.20$$

- ▶ For fully rough turbulent flow

$$\frac{1}{4\sqrt{f_c}} \approx \log_{10} \frac{z_r}{z_0}$$

# Wave Current Shear Stress

- ▶ The wave shear stress is defined as  $\tau_{wm}$  where  $f_w$  is the wave friction faction and  $u_{bm}$  is the bed velocity.

$$\tau_{wm} = \frac{1}{2} f_w \rho u_{bm}^2$$

$$\frac{1}{4\sqrt{4f_w}} + \log_{10} \frac{1}{4\sqrt{4f_w}} = \log_{10} \sqrt{\frac{RE}{50}} - 0.17 + 0.06 (4\sqrt{4f_w}) \quad \text{Smooth fully turbulent}$$

$$f_w = \frac{2}{\sqrt{RE}} \quad \text{Laminar flow}$$

$$RE = \frac{u_{bm} A_{bm}}{\nu} \quad A_{bm} = \frac{u_{bm}}{\omega}$$

# Wave Current Shear Stress

- ▶ The wave shear stress is defined as  $\tau_{wm}$  where  $f_w$  is the wave friction faction and  $u_{bm}$  is the bed velocity.

$$\tau_{wm} = \frac{1}{2} f_w \rho u_{bm}^2$$

Rough fully turbulent

$$\frac{1}{4\sqrt{f_w}} + \log_{10} \frac{1}{4\sqrt{f_w}} = \log_{10} \frac{A_{bm}}{k_n} - 0.17 + 0.24 (4\sqrt{f_w})$$

$$A_{bm} = \frac{u_{bm}}{\Omega}$$



# Phase angle

- ▶ The maximum shear stress does not occur when the wave velocity is highest
- ▶ In fact there is a phase shift

$$\tan \varphi = \frac{\frac{\pi}{2}}{\ln \frac{\kappa U_{*wm}}{z_o \omega} - 1.15}$$

- ▶ The boundary layer thickness can be defined as

$$\delta_w = \frac{\kappa U_{*wm}}{\omega} = \kappa \sqrt{\frac{f_w}{2}} A_{bm}$$

# Wave-Current Combined

- ▶ Assume the maximum combined shear stress is a function of the current shear stress and maximum wave shear stress

$$\tau_m = |\tau_{wm} + \tau_c|$$

- ▶ Determine which is dominate, the current or the wave component

$$\mu = \frac{\tau_c}{\tau_{wm}} = \frac{U_{rc}^2}{U_{*wm}^2}$$

$$C_\mu = \sqrt{1 + 2\mu \cos \phi_{wc} + \mu^2}$$

# Wave Current Combined

- ▶ For smooth turbulent boundary

$$\frac{1}{4 \sqrt{\frac{4f_{cw}}{C_\mu}}} + \log_{10} \frac{1}{4 \sqrt{\frac{4f_{cw}}{C_\mu}}} = \log_{10} \sqrt{\frac{C_\mu^2 RE}{50}} - 0.17 + 0.06 \left( 4 \sqrt{\frac{4f_{cw}}{C_\mu}} \right)$$

- ▶ For rough turbulent boundary

$$\frac{1}{4 \sqrt{\frac{f_{cw}}{C_\mu}}} + \log_{10} \frac{1}{4 \sqrt{\frac{f_{cw}}{C_\mu}}} = \log_{10} \frac{C_\mu A_{bm}}{k_n} - 0.17 + 0.24 \left( 4 \sqrt{\frac{f_{cw}}{C_\mu}} \right)$$

# Wave Current Combined

- ▶ The maximum shear stress can be calculated with the following expressions

$$\frac{\tau_{wm}}{\rho} = u_{*wm}^2 = \frac{1}{2} f_{cw} u_{bm}^2 \quad \text{Maximum Wave Shear}$$

$$\frac{\tau_m}{\rho} = u_{*m}^2 = C_{\mu} u_{*wm}^2 \quad \text{Maximum Shear}$$

# Wave Current Combined

- ▶ The wave acts as a roughness element for the current

$$\delta_{cw} = \frac{\kappa u_{*m}}{\omega}$$

- ▶ And the current shear velocity  $u_{*c} = (\tau_c / \rho)^{1/2}$  can be found

$$u_{*c} = u_{*m} \frac{\ln \frac{z_r}{\delta_{cw}}}{\ln \frac{z_0}{\delta_{cw}}} \left[ -\frac{1}{2} + \sqrt{\frac{1}{4} + \kappa \frac{u_c(z_r)}{u_{*m}} \frac{\ln \frac{\delta_{cw}}{z_0}}{\left( \ln \frac{z_r}{\delta_{cw}} \right)^2}} \right]$$