Engineering With Nature

Lecture 7

Goals of Today's Lecture

- Shear Stress Calculations for Currents
- Shear Stress Calculation for Waves
- Shear Stress Calculation for Combined Currents and Waves

Defining the Boundary Layer– Currents

- The current velocity at any distance from the bed, z, (free stream) can be defined by the current shear velocity, u_{*c}, κ, and z_o the distance above the bed where the velocity is zero.

$$u_c = \frac{u_{*c}}{\kappa} \ln\frac{z}{z_o}$$

- The shear velocity, u_{*c}, is defined by the equation where τ_c is the current shear stress and ρ is the fluid density.

$$u_{*c} = (\tau_c / \rho)^{1/2}$$

Defining z_o

- z_o can be defined by the following equations where ν is the viscosity and k_n is the equivalent sand grain roughness

$$z_0 = \begin{cases} \dfrac{\nu}{9u_*} & \text{for smooth turbulent flow} \\ \dfrac{k_n}{30} & \text{for fully rough turbulent flow} \end{cases}$$

Smooth or Rough Turbulent

- Determining the flow assumes you know the shear velocity. Thus solving these equations is an iterative process

$$\frac{k_n \; u_*}{\nu} \geq 3.3 \quad \text{for fully rough turbulent flow}$$

$$\frac{k_n \; u_*}{\nu} \leq 3.3 \quad \text{for fully smooth turbulent flow}$$

Shear Stress – Current

- The shear stress is often defined as where z_r is the reference elevation

$$\tau_c = \frac{1}{2} f_c \rho (u_c(z_r))^2$$

- Where the friction factor is defined as
- for fully smooth turbulent flow

$$\frac{1}{4\sqrt{f_c}} + \log_{10} \frac{1}{4\sqrt{f_c}} = \log_{10} \frac{z_r u_c(z_r)}{\nu} + 0.20$$

- For fully rough turbulent flow

$$\frac{1}{4\sqrt{f_c}} \approx \log_{10} \frac{z_r}{z_0}$$

Wave Current Shear Stress

- The wave shear stress is defined as τ_{wm} where f_w is the wave friction faction and u_{bm} is the bed velocity.

$$\tau_{wm} = \frac{1}{2} f_w \rho u_{bm}^2$$

$$\frac{1}{4\sqrt{4f_w}} + \log_{10}\frac{1}{4\sqrt{4f_w}} = \log_{10}\sqrt{\frac{RE}{50}} - 0.17 + 0.06\left(4\sqrt{4f_w}\right)$$ Smooth fully turbulent

$$f_w = \frac{2}{\sqrt{RE}}$$ Laminar flow

$$RE = \frac{u_{bm} A_{bm}}{\nu} \qquad A_{bm} = \frac{u_{bm}}{\omega}$$

Wave Current Shear Stress

- The wave shear stress is defined as τ_{wm} where f_w is the wave friction faction and u_{bm} is the bed velocity.

$$\tau_{wm} = \frac{1}{2} f_w \rho u_{bm}^2$$

Rough fully turbulent

$$\frac{1}{4\sqrt{f_w}} + \log_{10} \frac{1}{4\sqrt{f_w}} = \log_{10} \frac{A_{bm}}{k_n} - 0.17 + 0.24 \left(4\sqrt{f_w}\right)$$

$$A_{bm} = \frac{u_{bm}}{\omega}$$

Phase angle

- The maximum shear stress does not occur when the wave velocity is highest
- In fact there is a phase shift

$$\tan \varphi = \frac{\frac{\pi}{2}}{\ln \frac{\kappa u_{*wm}}{z_o \omega} - 1.15}$$

- The boundary layer thickness can be defined as

$$\delta_w = \frac{\kappa u_{*wm}}{\omega} = \kappa \sqrt{\frac{f_w}{2}} A_{bm}$$

Wave-Current Combined

- Assume the maximum combined shear stress is a function of the current shear stress and maximum wave shear stress

$$\tau_m = |\tau_{wm} + \tau_c|$$

- Determine which is dominate, the current or the wave component

$$\mu = \frac{\tau_c}{\tau_{wm}} = \frac{u_{*c}^2}{u_{*wm}^2} \qquad C_\mu = \sqrt{1 + 2\mu \cos \varphi_{wc} + \mu^2}$$

Wave Current Combined

- For smooth turbulent boundary

$$\frac{1}{4\sqrt{\frac{4f_{cw}}{C_\mu}}} + \log_{10}\frac{1}{4\sqrt{\frac{4f_{cw}}{C_\mu}}} = \log_{10}\sqrt{\frac{C_\mu^2\,\mathrm{RE}}{50}} - 0.17 + 0.06\left(4\sqrt{\frac{4f_{cw}}{C_\mu}}\right)$$

- For rough turbulent boundary

$$\frac{1}{4\sqrt{\frac{f_{cw}}{C_\mu}}} + \log_{10}\frac{1}{4\sqrt{\frac{f_{cw}}{C_\mu}}} = \log_{10}\frac{C_\mu A_{bm}}{k_n} - 0.17 + 0.24\left(4\sqrt{\frac{f_{cw}}{C_\mu}}\right)$$

Wave Current Combined

- The maximum shear stress can be calculated with the following expressions

$$\frac{\tau_{wm}}{\rho} = u_{*wm}^2 = \frac{1}{2} f_{cw} u_{bm}^2$$

Maximum Wave Shear

$$\frac{\tau_m}{\rho} = u_{*m}^2 = C_\mu u_{*wm}^2$$

Maximum Shear

Wave Current Combined

- The wave acts as a roughness element for the current

$$\delta_{cw} = \frac{\kappa u_{*m}}{\omega}$$

- And the current shear velocity $u_{*c} = (\tau_c / \rho)^{1/2}$ can be found

$$u_{*c} = u_{*m} \frac{\ln \frac{z_r}{\delta_{cw}}}{\ln \frac{\delta_{cw}}{z_0}} \left[-\frac{1}{2} + \sqrt{\frac{1}{4} + \kappa \frac{u_c(z_r)}{u_{*m}} \frac{\ln \frac{\delta_{cw}}{z_0}}{\left(\ln \frac{z_r}{\delta_{cw}} \right)^2}} \right]$$